Ad Soyad: 19.06.2021

Numara:

MATRİSLER TEORİSİ FİNAL SINAVI SORULARI

1. Üç tane elemanter matris yazınız, terslerini bulunuz.

2.
$$\begin{aligned} x+2y-z&=4 \\ -2x+3y-z&=1 \\ -2x-y+z&=-3 \end{aligned}$$
lineer denklem sisteminin çözümünü LU ayrışımı yardımıyla yapınız. (Diğer yöntemlerle çözüm kabul edilmeyecektir.)

3. $A=\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 3 \\ -1 & 3 & 4 \end{bmatrix}$ matrisinin tersini bulunuz.

4. $A=\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ matrisinin karakteristik değerlerini ve karakteristik vektörlerini bulunuz.

5.
$$\begin{aligned} -2x+y+z&=5 \\ x-2y+z&=-2 \\ x+y-2z&=-3 \end{aligned}$$
lineer denklem sistemini ilaveli asli determinantlar yardımıyla çözünüz.(Diğer yöntem çözümleri kabul edilmeyecektir.)

Matrisler Teo Final Sınavı Cevap Anahtarı

1) Elemanter matris; birim matrise sadece bir tane elemanter işlem uygulanması ile elde edilen matrise denir. Örneğin

$$I_2=\begin{array}{c|l}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} & \begin{array}{l}-\alpha_1\\-\alpha_2\end{array}\end{array} \quad E=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} \qquad \varepsilon_1:\alpha_1\rightarrow\alpha_1+\alpha_2$$

Tersi için $\varepsilon_1^{-1}:\ \alpha_1\longrightarrow\alpha_1-\alpha_2$ olur.

$$E^{-1}=\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$$ bulunur.

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2)

$$A=\begin{array}{ccc|l}\beta_1 & \beta_2 & \beta_3 & \\ 1 & 2 & -1 & -\alpha_1\\ -2 & 3 & -1 & -\alpha_2\\ -2 & -1 & 1 & -\alpha_3\end{array}$$

$\varepsilon_1:\ \alpha_2\rightarrow\alpha_2+2\alpha_1$

$\alpha_3\rightarrow\alpha_3+2\alpha_1$

$\varepsilon_2:\ \alpha_3\rightarrow\alpha_3-\frac{3}{7}\alpha_2$

$$A\overset{\varepsilon_1}{\sim}\begin{bmatrix}1 & 2 & -1\\0 & 7 & -3\\0 & 3 & -1\end{bmatrix}\overset{\varepsilon_2}{\sim}\begin{bmatrix}1 & 2 & -1\\0 & 7 & -3\\0 & 0 & 2/7\end{bmatrix}=U$$

$\varepsilon_3:\ \beta_2\rightarrow\beta_2-2\beta_1$

$\beta_3\rightarrow\beta_3+\beta_1$

$\varepsilon_4:\ \beta_2\rightarrow\frac{1}{7}\beta_2$

$$A\overset{\varepsilon_3}{\sim}\begin{bmatrix}1 & 0 & 0\\-2 & 7 & -3\\-2 & 3 & -1\end{bmatrix}\overset{\varepsilon_4}{\sim}\begin{bmatrix}1 & 0 & 0\\-2 & 1 & -3\\-2 & 3/7 & -1\end{bmatrix}\overset{\varepsilon_5}{\sim}\begin{bmatrix}1 & 0 & 0\\-2 & 1 & 0\\-2 & 3/7 & 2/7\end{bmatrix}$$

$$\overset{\varepsilon_6}{\sim}\begin{bmatrix}1 & 0 & 0\\-2 & 1 & 0\\-2 & 3/7 & 1\end{bmatrix}=L$$

$A=LU$ yazılır.

$AX=B$ denkleminde $A=LU$ yazılırsa

$(LU)X=B \Rightarrow L(UX)=B$ $\quad$ $UX=Z$ dersek.

$LZ=B$ olur.

$$\begin{bmatrix}1&0&0\\-2&1&0\\-2&3/7&1\end{bmatrix}\begin{bmatrix}z_1\\z_2\\z_3\end{bmatrix}=\begin{bmatrix}4\\1\\-3\end{bmatrix}$$

$z_1=4$

$-2z_1+z_2=1 \Rightarrow z_2=9$

$-2z_1+\frac{3}{7}z_2+z_3=-3$

$-8+\frac{\overset{35}{27}}{7}\overset{-5}{+3}=-z_3 \Rightarrow z_3=\frac{8}{7}$.

$UX=Z$ kullanılırsa

$$\begin{bmatrix}1&2&-1\\0&7&-3\\0&0&2/7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\9\\8/7\end{bmatrix}$$

$x+2y-z=4$

$7y-3\underset{4}{z}=9$

$\frac{2}{7}z=\frac{8}{7}$

$\to x=\underline{2}$

$y=3$

$z=4$

Kontrol edilen soru

3) $$[A:I_3]=\left[\begin{array}{ccc|ccc}1&1&0&1&0&0\\2&1&3&0&1&0\\-1&3&4&0&0&1\end{array}\right]\begin{array}{l}-\alpha_1\\-\alpha_2\\-\alpha_3\end{array}$$

$$\overset{\varepsilon_1}{\sim}\left[\begin{array}{ccc|ccc}1&1&0&1&0&0\\0&-1&3&-2&1&0\\0&4&4&1&0&1\end{array}\right]\overset{\varepsilon_2}{\sim}\left[\begin{array}{ccc|ccc}1&1&0&1&0&0\\0&1&-3&2&-1&0\\0&4&4&1&0&1\end{array}\right]$$

$$\overset{\varepsilon_3}{\sim}\left[\begin{array}{ccc|ccc}1&0&3&-1&1&0\\0&1&-3&2&-1&0\\0&0&16&-7&4&1\end{array}\right]\overset{\varepsilon_4}{\sim}\left[\begin{array}{ccc|ccc}1&0&3&-1&1&0\\0&1&-3&2&-1&0\\0&0&1&-\frac{7}{16}&\frac{1}{4}&\frac{1}{16}\end{array}\right]$$

$$\overset{\varepsilon_5}{\sim}\left[\begin{array}{ccc|ccc}1&0&0&\frac{5}{16}&\frac{1}{4}&-\frac{3}{16}\\0&1&0&\frac{11}{16}&-\frac{1}{4}&\frac{3}{16}\\0&0&1&-\frac{7}{16}&\frac{1}{4}&\frac{1}{16}\end{array}\right]=[I_3:A^{-1}]$$

$\underbrace{\qquad\qquad}_{A^{-1}}$

(Kontrol edilen soru)

4) $A=\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$

Karakteristik değerlerini bulalım.

$P_A(\lambda)=0 \qquad \det(\lambda I_3 - A)=0$

$$\left|\begin{matrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{matrix}\right| - \left|\begin{matrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{matrix}\right| = 0$$

$$\left|\begin{matrix} \lambda-3 & -1 & 1 \\ -2 & \lambda-2 & 1 \\ -2 & -2 & \lambda \end{matrix}\right| = \lambda^3-5\lambda^2+8\lambda-4=(\lambda-1)(\lambda-2)^2=0$$

$\lambda_1=1, \quad \lambda_2=2, \quad \lambda_3=2$

Karakteristik vektörleri bulalım.

$\lambda_1=1$ için $\quad A(\alpha)=\lambda_1\alpha$

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \qquad \begin{matrix} 3\alpha_1+\alpha_2-\alpha_3=\alpha_1 \\ 2\alpha_1+2\alpha_2-\alpha_3=\alpha_2 \\ 2\alpha_1+2\alpha_2=\alpha_3 \end{matrix}$$

$2\alpha_1+\alpha_2-\alpha_3=0$

$-/ \ 2\alpha_1+\alpha_2-\alpha_3=0$

$\underline{\quad 2\alpha_1+2\alpha_2-\alpha_3=0 \quad}$

$\alpha_2=0 \qquad 2\alpha_1=\alpha_3$

$, \ \alpha=\begin{bmatrix} t \\ 0 \\ 2t \end{bmatrix}=t\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

$\lambda_1=1$ için $\quad \alpha_1=\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ ve $\quad \alpha_2=\begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$ alınabilir.

$\lambda_3=2$ için $\quad A(\alpha)=\lambda_3\alpha$

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 2\alpha_1 \\ 2\alpha_2 \\ 2\alpha_3 \end{bmatrix}$$

$$\left.\begin{matrix} 3\alpha_1+\alpha_2-\alpha_3=2\alpha_1 & \Rightarrow & \alpha_1+\alpha_2-\alpha_3=0 \\ 2\alpha_1+2\alpha_2-\alpha_3=2\alpha_2 & \Rightarrow & 2\alpha_1-\alpha_3=0 \\ 2\alpha_1+2\alpha_2=2\alpha_3 & \Rightarrow & 2\alpha_1+2\alpha_2-2\alpha_3=0 \end{matrix}\right\} \text{aynı}$$

$2\alpha_1=\alpha_3 \qquad \alpha_1+\alpha_2=\alpha_3$

$\alpha_1=t \ , \ \alpha_3=2t \quad \alpha_2=t$

$\alpha=t\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ karakteristik vektörleri olur.

5) $A=\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

rankını bulalım.

$$\overset{\mathcal{E}_1}{\sim}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}\overset{\mathcal{E}_2}{\sim}\begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix}\overset{\mathcal{E}_3}{\sim}\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}\overset{\mathcal{E}_4}{\sim}\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\text{rank}A=2$

Asli det 2x2 tipinde determinantı sıfırdan farklı olan matrisler

$\delta_2=\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix}=3\neq 0$

İlaveli asli determinantı $\delta_{2+1}=\begin{vmatrix} -2 & 1 & 5 \\ 1 & -2 & -2 \\ 1 & 1 & -3 \end{vmatrix}=0$ olup

3-2=1 parametreye bağlı sonsuz çözüme vardır.

$$\begin{aligned} -2x+y+z&=5 \\ x-2y+z&=-2 \end{aligned}$$, $z=t$ için

Burada katsayıları δ_2 dan sistem cramerdir.

$$\begin{aligned} -2x+y&=5-t \\ x-2y&=-2-t \end{aligned}$$ cramer sistem

$$x=\frac{\begin{vmatrix} 5-t & 1 \\ -2-t & -2 \end{vmatrix}}{\delta_2}=\frac{3t-8}{3}\ ,\quad y=\frac{\begin{vmatrix} -2 & 5-t \\ 1 & -2-t \end{vmatrix}}{3}=\frac{3t-1}{3}$$

$z=t$

sistemin çözümüdür.